OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4755/01

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

MONDAY 14 MAY 2018: Afternoon

TIME ALLOWED: 1 hour 30 minutes plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book sent with the standard paper or any suitable paper supplied by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4755/01 sent with the standard paper
MEI Examination Formulae and Tables (MF2) sent with the standard paper

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (JM) 164837/1

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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SECTION A (36 marks)

1 The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = \begin{pmatrix} 2 & 2k & -k \\ 0 & 1 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 3 & -3 \\ -2 & 4 \end{pmatrix}$, where k is a constant.

(i) Find, in terms of k, the matrix $\mathbf{AB}$. [2]

(ii) Find the value of k for which matrix $\mathbf{AB}$ is singular. [2]

2 The quadratic equation $x^2 + px + q = 0$ has roots α and β, where

$$\alpha^2 + \beta^2 = -16,$$
$$\alpha - \beta = 6\mathrm{j}.$$

By considering $(\alpha - \beta)^2$, find the value of $\alpha\beta$. Hence state the value of q and find the possible values of p. [5]

3 (i) Sketch on an Argand diagram the set of points representing complex numbers z for which $|z - (3 + 3\mathrm{j})| = 3$. [2]

(ii) Find the greatest possible value of $|z|$ for this set of points. [2]

(iii) Mark on your Argand diagram the particular point for which $\arg(z - (3 + 3\text{j})) = \frac{2}{3}\pi$. Find this value of z in the form $a + \text{j}b$. [3]

4 **(i)** Use standard series formulae to show that

$$\sum_{r=1}^{n} r(2 + 3r) = \frac{1}{2}n(n + 1)(2n + 3). \quad [4]$$

(ii) Hence find the value of n such that

$$\sum_{r=1}^{4n} r(2 + 3r) = 198n(4n + 1). \quad [3]$$

5 You are given that $z = 2 + 5\text{j}$ is a root of the cubic equation $2z^3 - 5z^2 + pz + q = 0$, where p and q are real constants. Find the values of p and q. [6]

6 Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} r2^r = 2[1 + (n - 1)2^n]. \quad [7]$$

SECTION B (36 marks)

7 A curve has equation $y = \dfrac{2x^2 - 5x - 3}{x^2 + x - 2}$.

(i) Find the values of x for which $y = 0$. [2]

(ii) Find the equations of the three asymptotes. [3]

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for

(A) large positive values of x,

(B) large negative values of x. [2]

(iv) Sketch the curve. [3]

(v) Solve the inequality $\dfrac{2x^2 - 5x - 3}{x^2 + x - 2} \geqslant 0$. [3]

8 You are given that $\dfrac{1}{2r-1} - \dfrac{1}{2r+3} = \dfrac{4}{(2r-1)(2r+3)}$ for all integers r.

(i) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+3)} = k - \frac{n+1}{(2n+1)(2n+3)},$$

stating the value of k. [6]

(ii) The sum of the infinite series

$$\frac{1}{(2(n+1)-1)(2(n+1)+3)}+\frac{1}{(2(n+2)-1)(2(n+2)+3)}+\frac{1}{(2(n+3)-1)(2(n+3)+3)}+\cdots$$

is $\frac{7}{195}$. Show that n satisfies $28n^2-139n-174=0$ and hence find the value of n. [5]

9 You are given that $\mathbf{M} = \begin{pmatrix} 4 & a \\ -6 & -2 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} -2 & 6 \\ -4a & -14 \end{pmatrix}$, where a is a real constant. Find the possible value(s) of a in each of the following cases.

(i) The point $(1, -2)$ is invariant under the transformation represented by matrix $\mathbf{M}$. [2]

(ii) $(\mathbf{NM}^{-1})^{-1}\mathbf{NM} = \mathbf{N}$. [4]

(iii) A triangle T_1 has an area of 9 square units. The triangle T_1 is transformed to triangle T_2 by the transformation represented by matrix $\mathbf{M}$. The area of triangle T_2 is 144 square units. [6]

END OF QUESTION PAPER